COMPARISON OF OPTIMISATION ALGORITHMS TO DETERMINE CONTROL STRATEGIES IN UDS

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RTC of **UDS**, Evaluation of Control Strategy, Determination of Control Strategy, Linear Optimisation, Non-linear Optimisation

1. Introduction

In combination with other sanitation measures **RTC** can improve the performances of **UDS**. Control strategies can be determined by optimisation algorithms. They have following characteristics:

- control objectives are expressed as a cost function, whose value is to be minimised
- static (i.e. maximal storage or transport capacity) and dynamic characteristics of the **UDS** are expressed in so called domain restrictions[1]

Because of limited computation capacity, a general formulation of constraints is incompatible with on-line implementation. Most of the previous studies refer to linear optimisation, where restrictions and cost functions are linear combinations of decision variables. Specific non-linear formulations can make the mathematical transposition more flexible. On the other hand, their cost coefficients are more difficult to determine and computation times increase. This paper describes a linear and non-linear optimisation module. Aim of the study is not to get the very best of each method in a specific case. It will rather be tried to specify the characteristics of each method by analysing and comparing their results.

2. Principles of optimisation

2.1 Linear optimisation

A linear optimisation problem is expressed as follows; "minimise the cost function $\mathbf{f_1(X)}$ under following conditions: $\mathbf{AX \le B}$ and $\mathbf{X \geq 0}$"

$$f_1(X) = \Sigma\, (\Sigma\, \alpha_i X_{ik}),\ \text{summation over } i=1,..,n \text{ and } k=1,..,N$$

- **n** : number of decision variables and **N** : number of decision steps
- α_i : cost coefficients related to decision variable No i
- **X** : x_{ik} represent the decision variables at a given time step
- **A** : matrix (does not depend on **X**) and **B** : vector (does not depend on **X**)

The conversion of the practical control problem into the numerical one requires a specific representation of the **UDS** and its behaviour. Basic descriptive elements are nodes and connections.

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[1] In reality, it is also possible to express specific constraints in the direct formulation of the cost function

- Each connection corresponds to a transport capacity (i.e. collectors, pumps). It has a maximal flow capacity and an initial value. Besides, the flow process in each transport element is characterised by a constant transport time.
- Each node corresponds to a storage capacity (m^3). It has a maximal volume capacity and an initial value. Besides, it satisfies the principle of continuity.

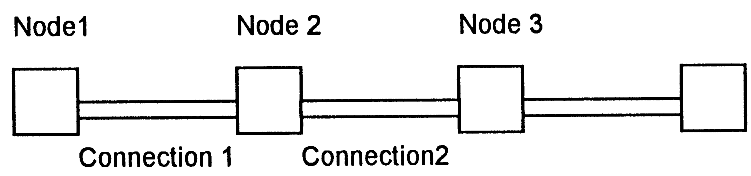

Fig. 1: simplified description of the **UDS** for optimisation

In a single calculation step for a given period of time [t,t+Ht], optimisation algorithms determine the values of the decision variables, which minimise the costs. **Ht** is an integer multiple of the decision interval, called horizon of optimisation. Theoretically, **Ht** should include the whole rainfall event. But the number of restrictions and decision values increases linearly with **Ht,** so that the computer capacity is rapidly overtaxed. In most of the cases, **Ht** only includes a few Δt. The optimisation successively calculates values for shifted horizon **Ht**, till the whole event is simulated.

The determination of the minimum of **$f_1(X)$** is based on the simplex algorithm developed by Dantzig (1948). Its description can be found in almost every book dealing with linear optimisation.

2.2 Non linear optimisation

In the non-linear optimisation the formulation of the domain restrictions remains unchanged, but the formulation of the cost function **$f_2(X)$** is different.

$$f_2(X) = \Sigma\ (\Sigma \alpha_i\ X_{ik}{}^{\beta_i} + \gamma_i(x_{iN}-x_{i1}))\ ,\quad i=1,..,n\ ,\ k=1,..,N$$

with α_i: cost coefficients, β_i form parameters and γ_i variability coefficients

The implemented algorithm is based on the "branch-and-bound-method" (see HORST, 1979 and ZOUTENDIJK,1960). The advantage of the method is that it can cope with convex elementary functions ($\alpha_i \geq 0$, $\beta i \geq 1$) and concave elementary functions ($\alpha_i \geq 0$, $\beta i < 1$). The characteristics of the system and its control can be accounted for in a more flexible way.

3. Study case

3.1 Description and Representations of the UDS

An artificial **UDS** has been selected (see **Fig. 2** and **Tab. 1**). It drains four identical catchments (E_i, i=1,4) of 100 ha each (50% impervious surfaces). The pipes cross sections are circular and their diameters range from 1m to 1.5 m. Besides the pipes storage capacity, the **UDS** contains five identical retention basins (Si, i=1,5). Each of them has a maximal capacity of 5000 m^3. There are five outlets.

Internal pump stations **D2** (upstream/downstream) , **D4** (upstream/downstream), **D5** (only downstream) control the transport. For every admissible flow direction, the maximal pumping capacity is 4 m^3/s, so that every retention basin can be filled and emptied in less than 25 minutes.

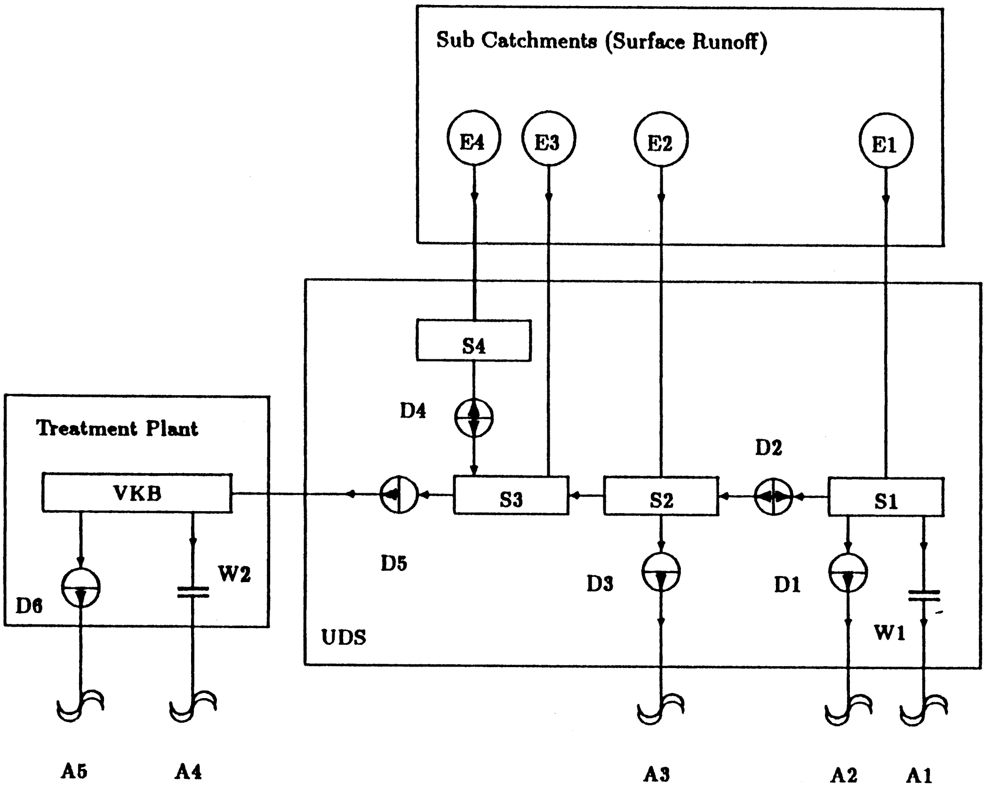

Fig. 2: System representation of the **UDS**

outlet	Type of connection	max. capacity Q	type of receiving waters
A1	Weir **W1**	> 60 m3/s	very sensitive to pollution loads
A2	Pump **D1**	4 m3/s	not very sensitive
A3	Pump **D3**	4 m3/s	not very sensitive
A4	Weir **W2**	> 110 m3/s	not very sensitive
A5	Pump **D6**	0.8 m3/s	Treatment plant

Tab. 1: System outlets

No	Notation	Type	stands for
1	**EZ1**	-	inflows from catchment **EZ1**
2	**EZ23**	-	inflows from catchment **EZ2+EZ3**
3	**EZ4**	-	inflows from catchment **EZ4**
1	**KV1**	internal node	Retention Bassin **S1**
2	**KV23**	internal node	Retention Bassins **S2+S3**
3	**KV4**	internal node	Retention Bassin **S4**
4	**KV5**	internal node	Retention Bassin **S5**
5	***KENT11***	*external node*	*Receiving Waters **F1***
6	***KENT12***	*external node*	*Receiving Waters **F2***
7	***KENT21***	*external node*	*Receiving Waters **F3***
8	***KENT51***	*external node*	*Receiving Waters **F4***
9	***KENT52***	*external node*	*Outlet of the TP*

Tab. 2: Description of the **UDS** for optimisation

No	Notation	Type	stands for
1	**BP1**	connection	**CSO** Pump
2	**BP2**	connection	internal Pump downstream
3	**BP3**	connection	internal Pump upstream
4	**BP4**	connection	**CSO** Pump
5	**BP5**	connection	internal Pump downstream
6	**BP6**	connection	internal Pump upstream
7	**BP7**	connection	internal Pump downstream
8	**BP8**	connection	**TP** treatment capacity
9	**BW1**	connection	Weir Outlet into **F1**
10	**BW2**	connection	Weir Outlet into **F4**

Tab. 2: Description of the **UDS** for optimisation

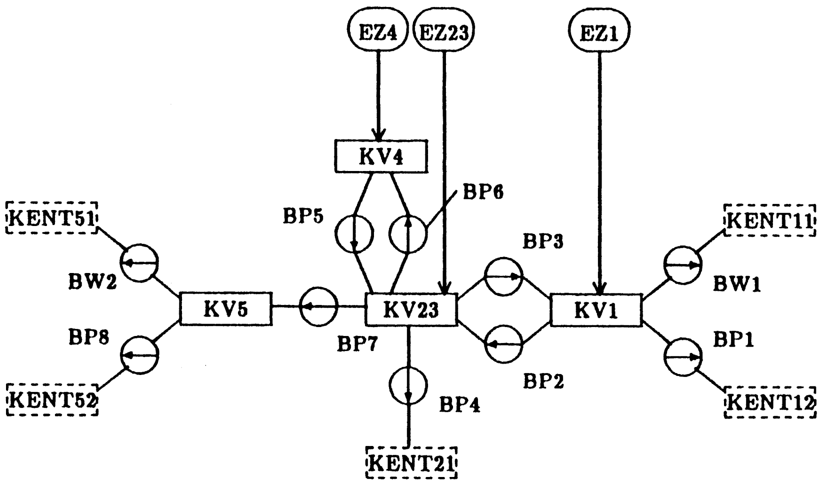

Fig. 3: Representation of the **UDS** for the optimisation programmes

3.2 The restrictions

Every decision variable must be non-negative. Supplementary restrictions are :

- non negative initial conditions in connections and nodes,
- non negative minimal and maximal capacity in connections (m^3/s) (14 inequalities)
- non negative minimal and maximal capacity in nodes (m^3) (10 inequalities)
- continuity equations in nodes (4 equalities + 5 equalities)

The total number of restrictions depends on **Ht** (see **Tab. 3**). A decision interval Δt is 5 min.

N	2	3	4	5
with external nodes	83	116	149	182
without external nodes	57	80	103	126

Tab. 3: Number of restrictions for various **N** (for the first optimisation horizon)

3.3 cost function (Tab. 4)

The costs were first determined for the linear optimisation **f1** according to following priorities: no flooding > no **CSO** > no storage > no transport. The cost function **f2** was determined on the basis of **f1**, as follows:

- costs coefficients for connections: α_i unchanged, β_i=2
- for internal nodes: β_i=2, coefficients α_i were calculated such that the storage costs remain higher than the transport costs
- for external nodes : β_i=0.2 and coefficients α_i were determined such that following inequality is verified; $\alpha_{KV}\, x_{KV}^2 \geq \alpha_{KENT} * 1\ m^3/\Delta t$, where KV designates the retention basin connected to the external node **KENT**

		linear optim.		non linear optim			
Element	max. cap. in m3	α_i	max. costs	β_i	α_i	γ_i	max. costs
BP1	**1200**	**1.00**	**1200**	**2**	**1.00**	**100**	**$1.44*10^6$**
BW1	**1950**	**1.00**	**1950**	**2**	**1.00**	**0**	**$3.80*10^6$**
BP2	1200	0.02	24	2	0.02	10	28800
BP3	1200	1.00	1200	2	1.00	10	$1.44*10^6$
BP4	**1200**	**1.00**	**1200**	**2**	**1.00**	**100**	**$1.44*10^6$**
BP5	1200	0.02	24	2	0.02	10	28800
BP6	1200	1.00	1200	2	1.00	10	$1.44*10^6$
BP7	1200	0.01	12	2	0.01	10	14400
BP8	240	0	0	0	0	50	0
BW2	**1950**	**1.00**	**1950**	**2**	**1.00**	**0**	**$3.8*10^6$**
KV1	5000	0.20	1000	2	0.08	0	$2.0*10^6$
KV23	10000	0.18	1800	2	0.03	0	$3.0*10^6$
KV4	5000	0.20	1000	2	0.08	0	$2.0*10^6$
KV5	5000	0.10	500	2	0.02	0	$5.0*10^5$
KENT11	10000	200.00	$2.0*10^6$	1/5	$2.0*10^6$	0	$1.26*10^7$
KENT12	10000	70.00	$7.0*10^5$	1/5	$1.7*10^6$	0	$1.07*10^7$
KENT21	10000	100.00	$1.0*10^6$	1/5	$2.5*10^6$	0	$1.57*10^7$
KENT51	15000	50.00	$7.5*10^5$	1/5	$0.6*10^6$	0	$4.11*10^7$
KENT52	35000	0.00	0	1/5	0	0	0

Tab. 4: cost functions for linear and non linear optimisations

Optimisations were also performed without external nodes (**KENT**). In this cases, the costs for corresponding external connections (**BP** resp. **BW**) were adjusted as follows; $\alpha_{BP}=\alpha_{KENT}$, $\beta_{BP}=1$, γ unchanged.

4. Some results

For several rainfall-runoff events optimisation results have been obtained, here a single event (**Tab. 5**) and its results (**Tab. 6**) are presented.

No.	Rainfall height (mm)	Rainfall duration (min)	Runoff duration (min)	runoff volume (m3)	max. inflow rate (m3/5min)	a_u (min)
1	15	30	104	30 000	2485	20

Tab. 5: rainfall and run-off characteristics of reference event

Obviously, something is wrong in the costs of the external nodes in the non-linear optimisation: the greater the optimisation horizon, the more **CSO** increases!. After the α_i coefficient for **KENT51** was doubled, the amount of **CSO** was reduced of 50%, but the deterioration of the strategy results for greater **N** remained. The concavity of the elementary cost function in **KENT51** (β_i =0.2), is responsible for this negative tendency.

If the external nodes are removed (=no concave elementary functions), no deterioration of the results occurs, when the horizon increases.

No.	Optim. method	external nodes	N	KENT51 m^3	KENT51 duration in Δt (-)	KENT51 max Q in m^3/5min
1	non linear	yes	2	4786	37	893
2	non-linear	yes	3	6186	34	614
3	non-linear	yes	4	8686	32	682
4	non-linear	yes	5	10788	30	699
5	non-linear	no	2-5	2500	7	893
6	linear	no	2-5	2513	7	893

Tab. 6: results of various optimisations for the reference rainfall-runoff event[2]

The amounts of **CSO** are almost the same for both optimisation methods and independent of the optimisation horizon. This is all the more astonishing, if we compare the computation times (**Tab. 7**).

Optimisation method	External nodes	N=2 (h.min)	N=3 (h.min)	N=4 (h.min)	N=5 (h.min)
non linear	yes	2.07	5.43	12.14	21.53
non linear	no	0.11	0.27	0.54	1.37
linear	yes	0.04	0.09	0.18	0.33
linear	no	0.02	0.04	0.10	0.15

Tab. 7: Comparison of the computation times

The utilisation of the storage capacity is almost independent of **N** in both methods. In the non-linear optimisation the distribution of the storage volumes in the different basins is more even. This is due to the quadratic form of the elementary cost function related to storage. The pump flow rates are also more regular in the non-linear optimisation, as long as the optimisation horizon remains small. But when **N** increases, the non-linear optimisation generates more and more irregular flow patterns. In linear optimisation the contrary is true . The negative tendency in the non-linear optimisation is due to the fact that small improvements in the exploitation of the storage capacity must be paid by a more and more erratic control of the pumps.

5. Conclusions

A linear and a non linear optimisation algorithm to determine control strategies in **UDS** have been briefly presented and applied in a study case. The following conclusions can be made:

- optimisation horizon does not seem to be a very sensible parameter
- concave elementary function should be carefully used, if ever
- linear optimisation gives as good results as non-linear optimisation, provided the amount of **CSO** is the predominant parameter of evaluation. But a non linear optimisation achieves a better exploitation of the available storage and transport capacity.

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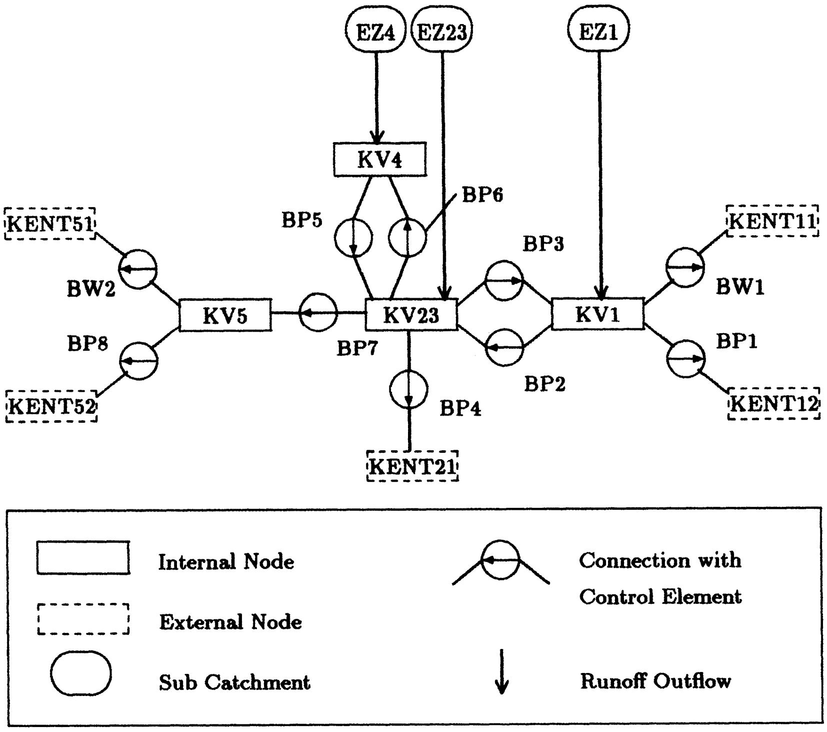
[2]For this event, no **CSO** in **KENT11**, **KENT12** and **KENT21** occurs.


EZ4
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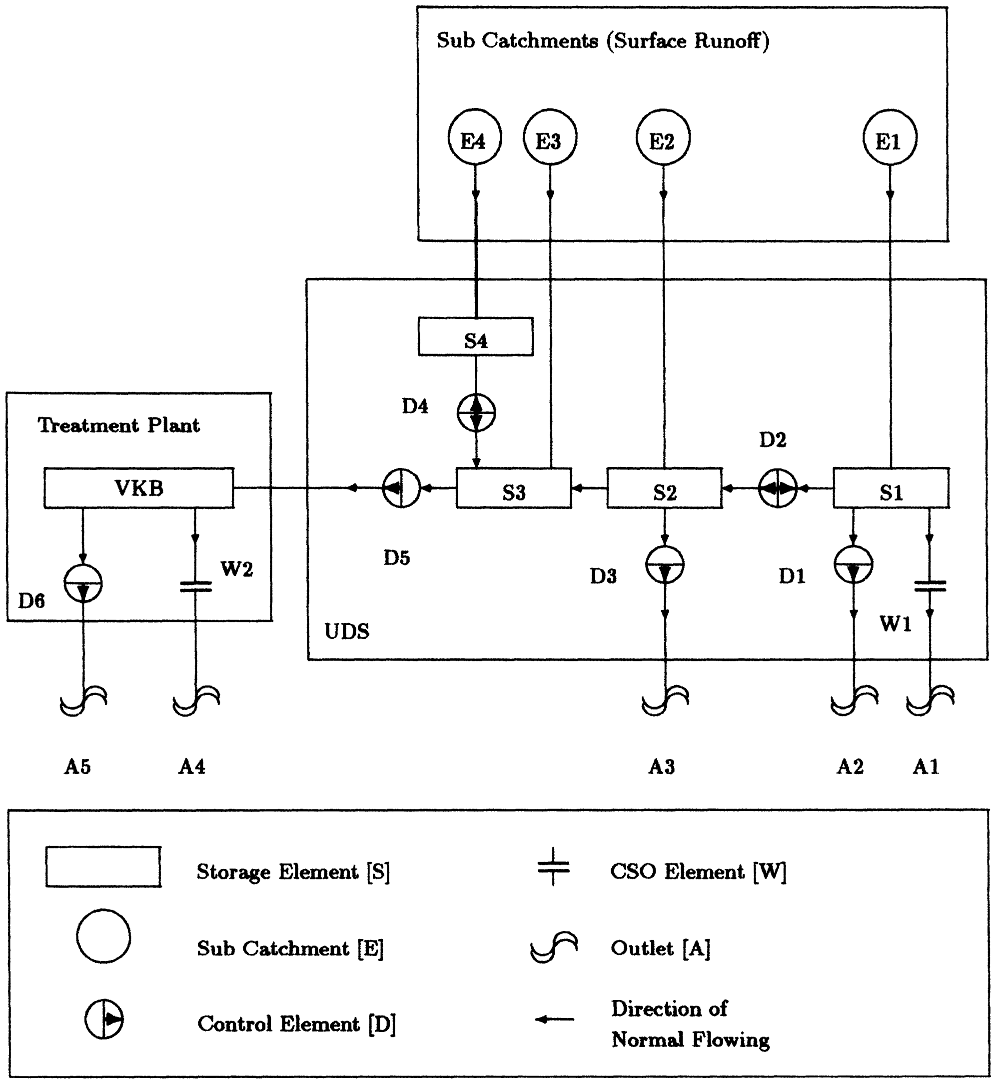
KENT21
Internal Node
External Node
Sub Catchment
Connection with
Control Element
Runoff Outflow


Sub Catchments (Surface Runoff)
E4
E3
E2
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Treatment Plant
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Outlet [A]
Control Element [D]
Direction of
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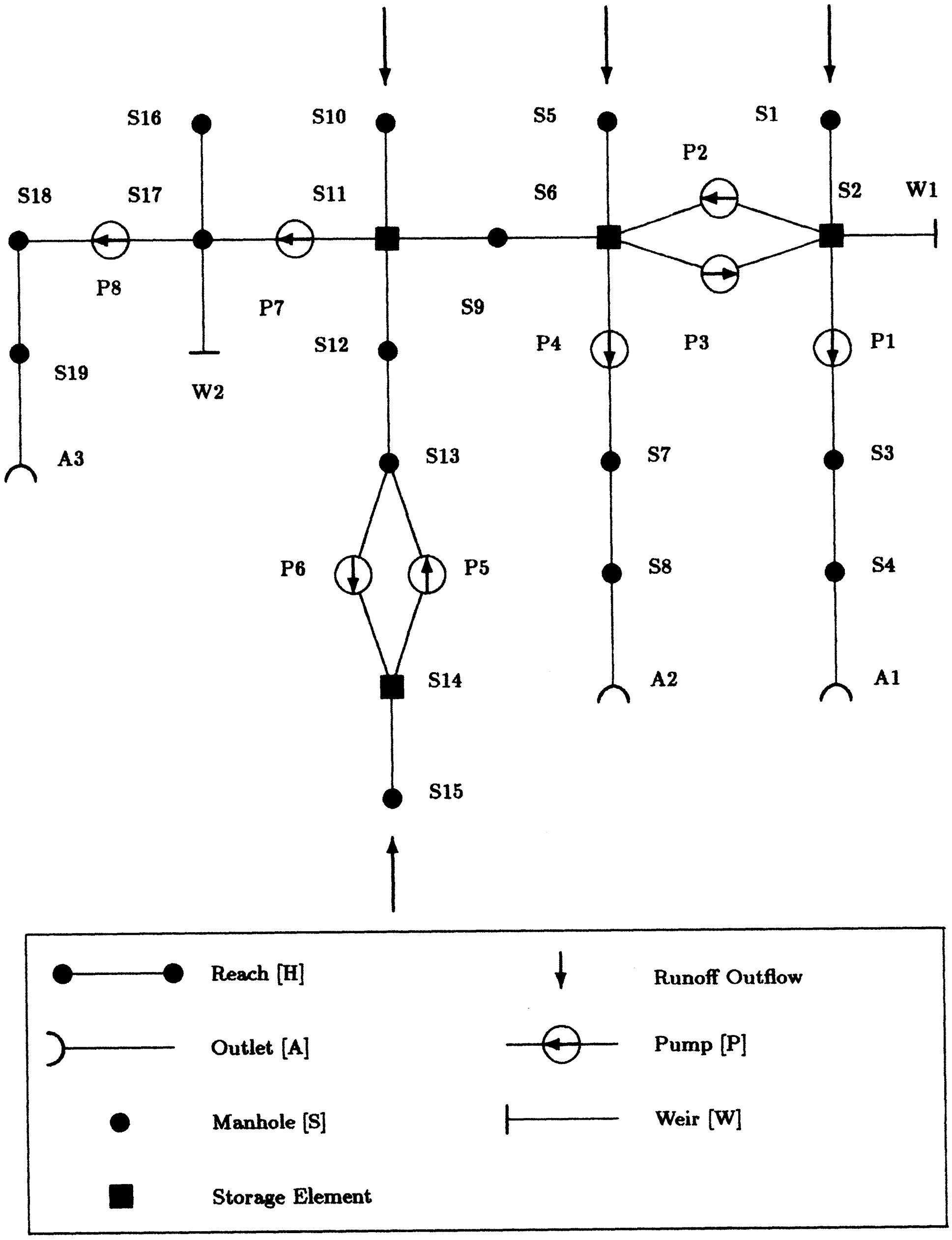
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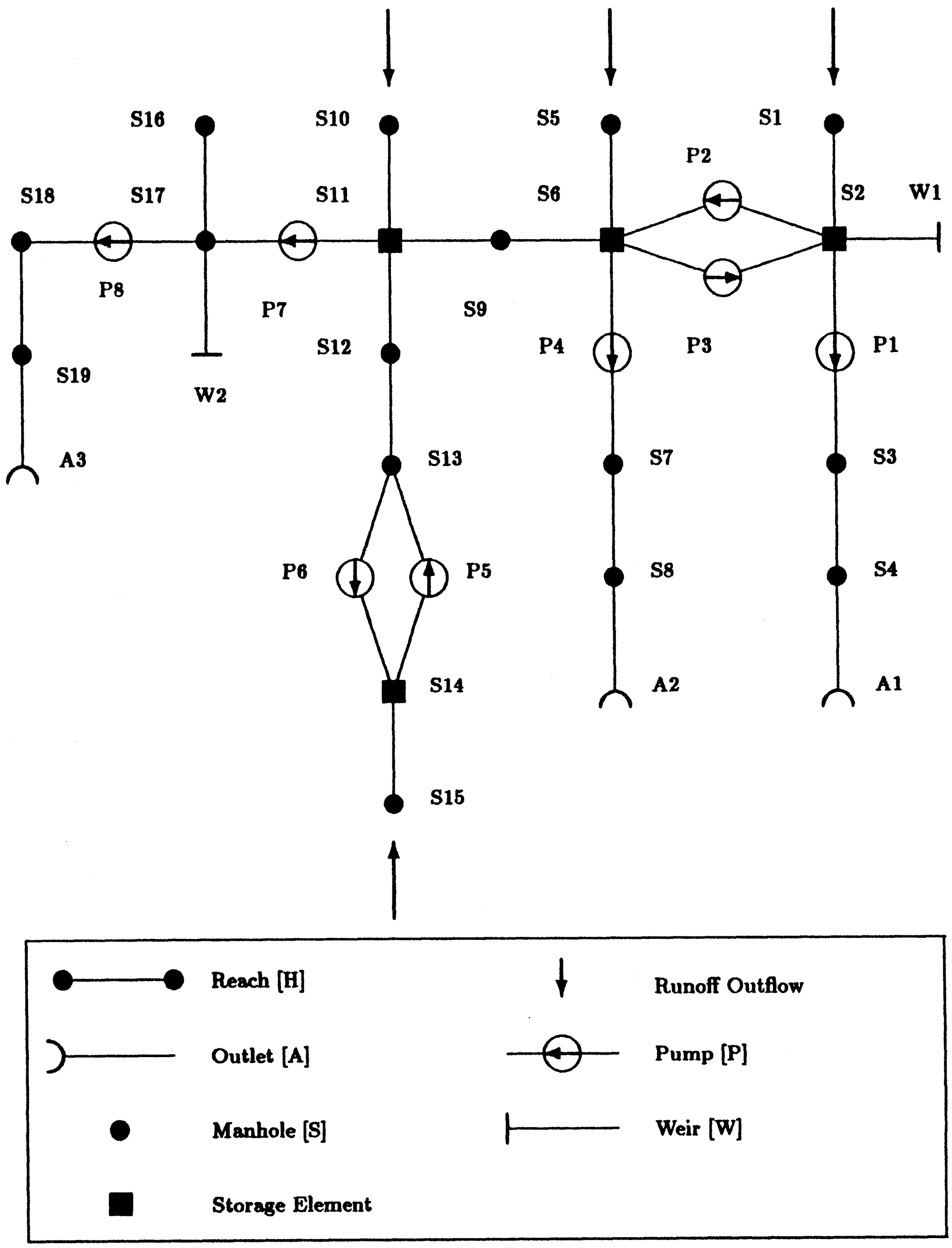
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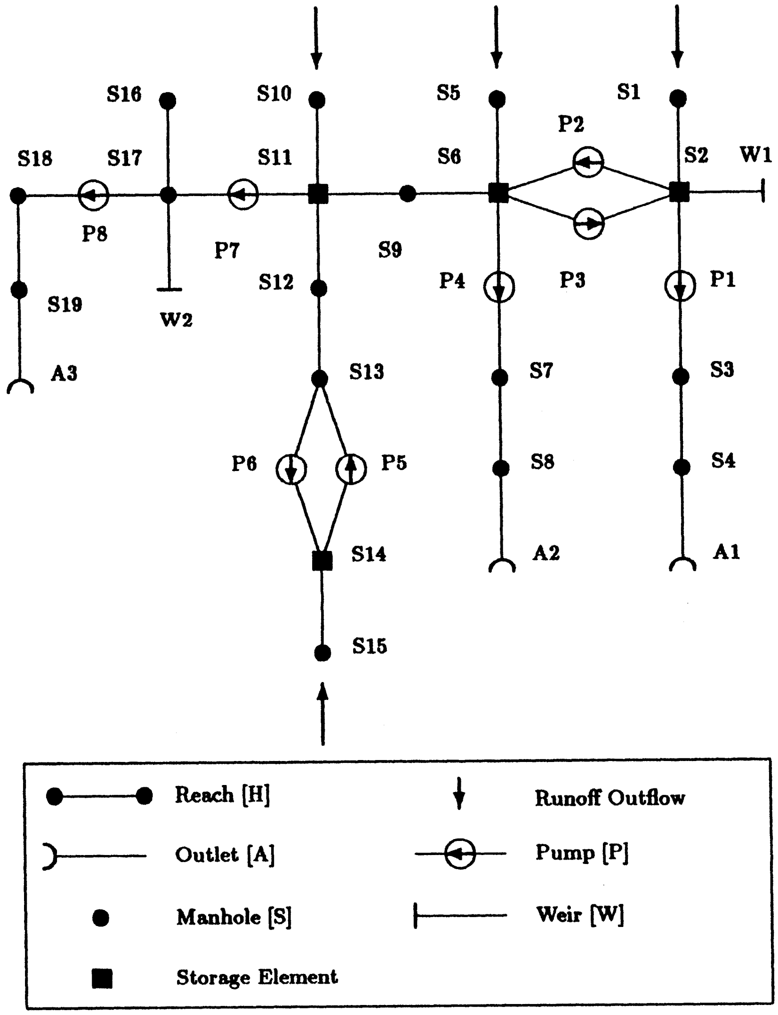
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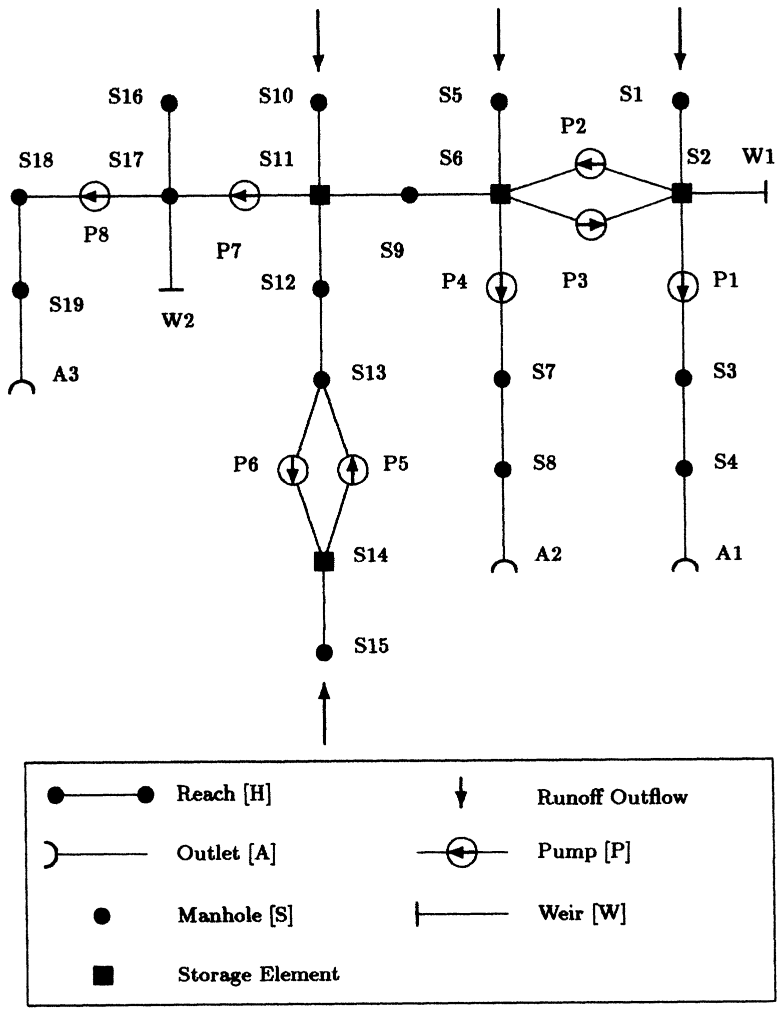
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