- Construction / Verification of simplified models on the basis of a detailled description of the UDS -

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The last developments of computer technology has made the use of advanced numeric models possible, to apprehend the behavior of Combined Urban Drainage Systems (CUDS). Advanced numeric models show following characteristics:

- the mathematical formulation of the flow processes in pipes bases on the equations of Barre-de-Saint-Venant (1872), which apply to unsteady flows in open channels.- A model extension is necessary to describe pressure flows, which in most models bases on the introduction of a virtual pipe slot, according to a proposition from Preismann. -
- the description of the UDS can be extremely detailled. Nowadays data bases are available on the german market, which in the best case can record every gully, and every real canalisation element. The complete UDS of a small town (ca. 60 000 inhabitants) includes around 6 000 records, each of which containing till 50 data units or more, depending on the type of model and calibration method.

As far as storage capacity and calculation velocity is concerned, personal computers are already able to very precisely simulate the entirely UDS of small cities for short periods (single rainfall events). It is not irrealistic to envisage that long-term detailled simulations will also be possible in the next years. In spite of these positive technological developments, the question of model simplifications remains worth of studying. This is especially the case, when developing modules for the determination of Real Time Control (RTC) strategy based on using optimization algorithms.

- Simplified modelling is included in the built-in simulation of the optimization algorithm (often based on iterative calculation steps according to the Newton-gradient-method).
- Simplified modelling may also be used to perform a short-time prediction of runoff inflows into the UDS, which may significantly improve the validity of optimization.

In the frame of a research project, whose general objective is to develop a RTC-System in the Water Division „Obere Iller“ (WDOI), South Bavaria, Germany, the university of Munich is developing und checking algorithms to automatically produce simplified models of the behavior of UDS. These models exclusively base on (detailled) network and catchment data. (No measurement data and/or results of a reference model is necessary). The proposed paper will shortly describe the theoretical basis of the involved simplified models, the proposed algorithms for simplification and the criteria for model validation and comparison. The methodology will be applied on the real case of Sonthofen, South Bavaria, which is a subcatchment under the supervision of the WDOI.

CONSTRUCTION OF SIMPLIFIED DESCRIPTIONS OF A SEWER SYSTEM TO CHARACTERIZE ITS HYDRAULIC BEHAVIOUR

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Introduction

Computing the flow processes in urban drainage systems with n elements by using hydrodynamic models is computing a system of $2n$ partial differential equations, that can be devided in n systems of 2 equations linked by their boundary conditions. Hence, computing the flow processes in urban drainage systems takes enormous numerical effort, what means a lot of computing time. If one's goal is to design a real time control strategy by using mathematical optimization methods, the flow process has to be computed several times, at least once every time you make a control decision. Furthermore, if the optimization scheme is iterative - it should be iterativ because of the nonlinearity of the drainage and the differential equation system - in every iteration step the flow processes have to be computed once. Also when using powerful mainframe computers or parallel machines the amount of computing time needed seems too large - even for off-line studies. Therefore studying methods to decrease computing time without loss of precission comes out to be a natural purpose for engineers and mathematicians. There have already been some studies on this topic. One way is using simplified equations like quasi-steady flow equations, uniform flow equations, kinematic waves or pure translation equations. It was shown, however (see *Schmitt*, 1985), that some important flow situations (e.g. backwater effects, flux deviding in meshed nets, upstream flow) can not be calculated correctly with these equations. Another way is to use simplified equations only where these problems do not appear and to use the original equations where they might appear. Doing this way has a major disadvantage: the computing algorithm has to be changed: either the user has to tell the programm where simplified equations should be applied or the program should recognize these parts itself. In the first case the behaviour of the sewer system must be known very well before starting the simulations, in the second case, automatic recognizing takes computing time in return.

A different approach to decrease computing time is to simplify the discription of the sewer system, rather than the simulation model, so that fewer elements must be computed. By doing this, usage of the old programms is still possible, because all you do is to simulate another - smaller - sewer system.

Methods Of Decreasing the Number of Sewer Elements

The following methods have been examined in this study:

(a) deleting all upstream elements with a diameter of less than a given d_{min}
(b) deleting all sewer elements upstream from a given element j_{ab}
(c) assigning a sequence of elements as a collector and delete every element connected to this collector
(d) deleting all elements j shorter than l_{del} and their upstream node $\pi(j)$ from the sewer system, and connect the elements that are linked to $\pi(j)$ to the downstream node $\sigma(j)$
(e) like method (d), but add the length of j to the length of an appropriate element j' up- or downstream from j; this should be done only if j and j' have the same diameter
(f) like method (e), but only if j and j' have the same slope and the same bottom levels
(g) a combination of any of the methods above

One must make sure not to remove hydraulic problems from the system nor to create nonexistent problems artificially by deleting elements. Therefore, the parameters j_{ab}, l_{del}, d_{min} and the collector of (c) should be chosen carefully.
Methods (a), (b), (c) change only upstream parts of the drainage system. Hence, the drainage area of the deleted elements must be added to the drainage area of new upstream nodes. In methods (d), (e), (f) the downstream configuration of the network has been altered, too. The drainage area of the deleted elements should be added to the drainage area of j'.
If l_{del} in method (d) is too large, the flow time within the network might be significantly reduced and there might be a loss of storage volume in the system. Therefore, l_{del} has been set to 5 m or less. With method (f), one can expect that none of the critical elements will be deleted; at the same time, it is possible that many noncritical elements will remain.
By deleting downstream elements, one must consider the fact that the bottom level and slope of the remaining elements must be changed, too. By using methods (a), (b), (c), the danger arises that the resulting drainage areas of the new upstream elements might be so large that the runoff model must be recalibrated.
The methods (a) and (d), (e), (f) run automatically, insofar as no further decision by the user is needed. In the methods (b) and (c) the user must decide which parts of the network should be removed.
Methods (d), (e), (f) are closely related to the hydraulic model EXTRAN, which solves the hydrodynamic equations by means of an explicit numerical scheme (see *Fuchs, et al.*, 1993). EXTRAN is a standard hydraulic model, from which many programs were derived. It has been assumed that erasing elements of small length would allow a longer calculation time-step. It has been observed, however, that many short elements are problematic in a hydraulic sense. Hence, there was only limited success in increasing the calculation time-step in EXTRAN.

Effectivness in Reducing the Number of Sewer Elements

Decreasing the number of sewer system elements saves computing time. The savings in computation time depend on the mathematical formulation of the hydrodynamic model as well as the numerical algorithms used to solve it.

All hydrodynamic models describe the fluid flow in a sewer element with conservation laws for mass and momentum. In a one-dimensional approach they are

$$\frac{\partial}{\partial t}\begin{pmatrix} A \\ u \end{pmatrix} + \begin{pmatrix} u & A \\ g\frac{\partial h}{\partial A} & u \end{pmatrix} \cdot \frac{\partial}{\partial x}\begin{pmatrix} A \\ u \end{pmatrix} = \begin{pmatrix} 0 \\ g(S_0 - S_f) \end{pmatrix} \tag{1}$$

Equation (1) is the Saint-Venant-equation system in matrix notation (see *Schmitz/ Edenhofer*, 1983). x and t are the independent variables, $A(x,t)$ and $u(x,t)$ are the dependent variables cross sectional area and average flow velocity, $h(x,A)$ is the water depth as a function of x and A (h is an *a priori* known function describing the geometry of the sewer element). S_0 is the slope of the sewer element, S_f is the friction slope (a function of A and u), that can be computed by the formulas of Prandtl-Colebrook or Manning-Strickler, which are not given here (see *Munson et al*, 1990). Equation (1) describes only free surface flow. For special treatment of other flow situations (pressure flow, pumps, storm-water outlets, storm-water-tanks and so on) other equations are used.

To solve the hyperbolic system (1) some initial conditions (2) and boundary conditions (3), given by appropriate Functions $\mathcal{F}_{1,2}$ are required. They read

$$u(x,0) = u_0(x) \quad \text{and} \quad A(x,0) = A_0(x) \tag{2}$$

$$\mathcal{F}_1\left(u(0,t), A(0,t)\right) = 0 \quad \text{and} \quad \mathcal{F}_2\left(u(\hat{x},t), A(\hat{x},t)\right) = 0 \tag{3}$$

where $\hat{x} = L$ is the downstream end of the sewer element for subcritical flow and $\hat{x} = 0$ the upstream end of the sewer element for supercritical flow. In fact, if the flow is supercritical, one can consider (1), (2), (3) as a pure initial value problem with a nonconstant initial curve. Equations (2) and (3) come from the theory of hyperbolic equations (see *Sauer*, 1952). In order to complete the mathematical model for a description of the flow processes in a sewer system, the functions $\mathcal{F}_{1,2}$ must be defined. Most models do not differentiate between subcritical and supercritical boundary conditions, what in fact would be a nontrivial and hard thing to do. They always assume subcritical boundary conditions. The boundary conditions are the connections of the sewer elements. For a node in which m elements converge $m+1$ internal boundary conditions are needed. From the water depth h^i in the node i come the m equations

$$h^i(t) = h^i_j(t) \tag{4}$$

where h^i_j is the water depth in sewer element j at node i. Once again special cases are not considered. The last boundary condition is the conservation law of mass in the node i. A possible formulation in terms of the discharge $Q = A \cdot u$ is

$$\frac{dV^i}{dt}(t) = Q^i(t) + \sum_j Q^i_j(t) \tag{5}$$

where Q^i is the runoff inflow to node i and Q^i_j is the inflow or outflow from element j to node i. V^i is the volume of the water in node i. To compute V^i the geometry of the node i must be known. Many calculation programs (especially EXTRAN-based programs) use a modified formulation of equation (5). They introduce a total storage area A^i_S of the node. A^i_S is a function of the elements connected to the node i. From V^i comes h^i since $dV^i/dt = A^i_S \cdot dh^i/dt$.
The amount of computing time needed to solve the equations (1) to (5) for a given sewer system depends on three major facts of the chosen numerical methods: first is the number of grid points in the sewer system, second is the length of the time step (explicit finite difference methods are constrained by the **C**ourant-**F**riedrich-**L**evy condition), third is the used scheme for solving the ordinary differential equation (5), where choosing an implicit scheme for (5) forces to choose an implicit model for equation (1).
If the number of grid points is constant for all elements (as it is in EXTRAN) the computing time increases nearly proportionally to the number of sewer elements for explicite schemes, that is $\mathcal{O}(n)$. If an implicite scheme for (5) is applied, it may be $\mathcal{O}(n^\alpha)$ with $\alpha > 1$. Hence, decreasing the number of sewer system elements has an effectiveness of order α in the implicit case and order 1 in the explicit case. If the number of grid points depends on the length of the corresponding element, deleting n_{del} short elements by methods (d) to (f) might be less efficient than deleting n_{del} elements by (a) to (c).

A Brief Discussion of Some Results

Case studies for two small town-sewer systems, A and B, will be summarized below. All simulations have been performed by an explicit EXTRAN-based model (see *Fuchs et al*, 1993, for description) for a single significant storm event. A full discussion is given in *Eberl*. Only methods (a) and (d), (e), (f) are used, because the effects and problems of (b) and (c) are the same as in (a), but they do not run automatically as mentioned above. Before running (e) and (f) with different l_{del} values, method (d) was used with l_{del} = 5m. In (a) different values for d_{min} were selected. The results are computed without recalibration of the runoff model. In drainage system A there is no combined sewage overflow outlet but five regular outlets (without throttles). In B there is just one regular sewer outlet (with a throttle) and some combined sewage overflow outlets. In the following tables the number n of the sewer elements is given. The hydraulic behaviour is described by the outflow function $Q(t)$. The following parameters are selected: the relative error ϵ_V in the total outflow from regular outlets, the relative error ϵ_T in the average flow time and the relative least-square error, ϵ_G, of the outflow. They are given by

$$\epsilon_V = \frac{\mid \int_0^T Q(t) - \hat{Q}(t)dt \mid}{\int_0^T \hat{Q}(t)dt} \tag{6}$$

$$\epsilon_T = \mid 1 - \frac{\int_0^T tQ(t)dt}{\int_0^T Q(t)dt} \frac{\int_0^T \hat{Q}(t)dt}{\int_0^T t\hat{Q}(t)dt} \mid \tag{7}$$

$$\epsilon_G = \frac{\int_0^T \left(Q(t) - \hat{Q}(t)\right)^2 dt}{\int_0^T \hat{Q}^2(t) dt} \tag{8}$$

where $\hat{Q}$ denotes the outflow of the unmodified systems. In *Tab.2* the largest time-step Δt that globally fullfills the CFL condition is also given. Method (a) does not increase the time-step in these cases.

	A	(a)	(a)	(a)	B	(a)	(a)	(a)
d_{min}	–	0.2m	0.3m	0.4m	–	0.2m	0.3m	0.4m
n	811	769	485	364	641	566	278	217
ϵ_V	–	0.0%	0.2%	0.5%	–	0.8%	1.3%	6.5%
ϵ_T	–	0	3.1%	4.6%	–	0.7%	0.1%	2.1%
ϵ_G	–	0.0%	0.16%	0.21%	–	0.26%	0.64%	1.46%

Table 1: results for method (a)

For drainage system A the results are good, in system B the simulation results are not quite as good at least for $d_{min} = 0.4$m. Analyzing the upstream parts of the sewer systems A,B with $d_{min} = 0.4$m, one can see, that in A 32% of the total drainage area is connected to 35 elements, while in B 41% is connected to 15 elements. Hence, a recalibration of the runoff parameters for the large drainage areas in the upstream parts of B should improve the results.

	A	(e)	(e)	(f)	B	(e)	(e)	(f)
l_{del}	–	20m	40m	80m	–	20m	40m	80m
n	811	691	530	713	641	535	408	574
ϵ_V	–	0.2%	0.4%	0.1%	–	2.7%	4.6%	0.5%
ϵ_T	–	6.5%	2.7%	0.3%	–	3.1%	17.7%	0.2%
ϵ_G	–	0.29%	0.08%	0.03%	–	0.25%	4.43%	0.08%
Δt	0.33s	0.54s	0.54s	0.33s	0.17s	0.27s	0.27s	0.17s

Table 2: results for methods (d), (e), (f)

Method (f) works well for both drainage systems, even with a large l_{del}, but the number of elements was only decreased by about 10%. In (e) there are differences between sewer systems A and B. Computing drainage system B according to method (e), greater errors were produced than in the other simulations. This can be explained by the theory of hyperbolic differential equations (see *Sauer*, 1952). It says, assuming the CFL-condition is met for an explicit finite-difference scheme (like EXTRAN), the discretisation error in a single element increases in function of its length (this comes from the local characteristic slopes of the partial differential equation (1)). Hence, since the globally defined calculation time-step Δt is determined by one of the short elements, the more long elements are in the drainage system the larger is the total amount of the discretisation error. In drainage system A lengths of the longest elements have not changed a lot, but in system B there are more long elements in the system after running (e). The greatest value shows a difference of more than 40m.

Tables 1 and 2 show, that - with exception of some problematic cases that are already explicitly discussed - the values describing the hydraulic behaviour of the simplified systems are very good. But what we can see from these results, too, is, that it might be dangerous to use the methods to decrease the number of sewer elements without investigation of its effects on the geometry of the sewer system. The behaviour of the maximum length of the elements in the methods (d), (e), (f) and the magnitude of the drainage area connected to the upstream elements can be used as indicators to judge the sewer system modification. But in the whole the discussed methods seem to work very good: for system *A* n could be decreased more than 50% with a single method and still very good results were reached. Combining the methods will improve the results.

Conclusion

Several methods to decrease the number of sewer elements, and, hence, to decrease computing time for the numerical analysis of sewer systems, have been introduced and shortly discussed. It was shown, that the efficiency of the methods in saving computing time depends also on the used computing model, e.g. in implicit schemes it will be greater than in explicit ones. There are also differences between methods which alter the sewer system in its downstream parts and those which alter the upstream parts. The latter ones will possibly require a recalibration of the runoff model. In the former ones problems in modifying the remaining network will arise. Until now, only the hydraulic behaviour of the sewer outlets was investigated. The subsequent step for the comparison of the methods should include other parameters, which further describe the hydraulic behaviour of a drainage system (characterization of the pressure flow frequencies, of flooding frequencies, etc).

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Die Schwerpunktlaufzeit t_L als Vergleichskriterium von Ganglinien ? (nicht Beurteilungskriterium einer einzigen Ganglinie!)

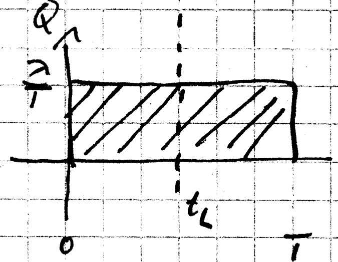
$$t_L := \frac{\int_0^T t\,Q(t)\,dt}{\int_0^T Q(t)\,dt}$$

I sei $Q_1(t) = \lambda Q_2(t)$, $\lambda \hat{=} const$

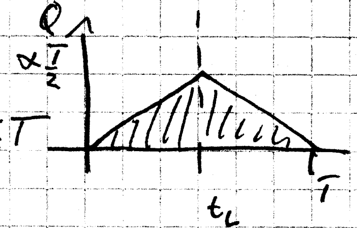
$$\Rightarrow t_{L,1} := \frac{\int_0^T t\,Q_1(t)\,dt}{\int_0^T Q_1(t)\,dt} = \frac{\lambda\int_0^T t\,Q_2(t)\,dt}{\lambda\int_0^T Q_2(t)\,dt} = \frac{\int_0^T t\,Q_2(t)\,dt}{\int_0^T Q_2(t)\,dt} =: t_{L,2}$$

II. sei $Q_1(t) := \lambda \cdot \frac{1}{T}$ für $0 \le t \le T$

$$\Rightarrow t_{L,1} := \frac{\int_0^T t\frac{\lambda}{T}\,dt}{\int_0^T \frac{\lambda}{T}\,dt} = \frac{\frac{T^2}{2}}{T} = \frac{T}{2}$$



• sei $Q_2(t) := \begin{cases} \alpha t & 0 \le t \le T/2 \\ \alpha\frac{T}{2} - \alpha(t-\frac{T}{2}) = \alpha T - \alpha t, & T/2 \le t \le T \end{cases}$

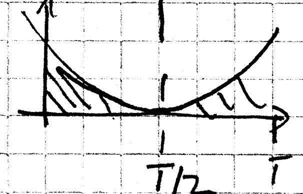


$$t_{L,2} := \frac{\int_0^{T/2} \alpha t^2\,dt + \int_{T/2}^T (\alpha T t - \alpha t^2)\,dt}{\int_0^{T/2} \alpha t\,dt + \int_{T/2}^T (\alpha T - \alpha t)\,dt} =$$

$$= \frac{\frac{1}{3}(\frac{T}{2})^3 + \frac{1}{2}T(T^2-(\frac{T}{2})^2) - \frac{1}{3}(T^3-(\frac{T}{2})^3)}{\frac{1}{2}(\frac{T}{2})^2 + T\cdot\frac{T}{2} - \frac{1}{2}(T^2-(\frac{T}{2})^2)} = \frac{\frac{1}{24}+\frac{1}{2}-\frac{1}{8}-\frac{1}{3}+\frac{1}{24}}{\frac{1}{8}+\frac{1}{2}-\frac{1}{2}+\frac{1}{8}} \cdot T$$

$$= \frac{4}{8}T = \frac{1}{2}T$$

• sei $Q_3(t) := \alpha(t-\frac{T}{2})^2$, $0 \le t \le T$

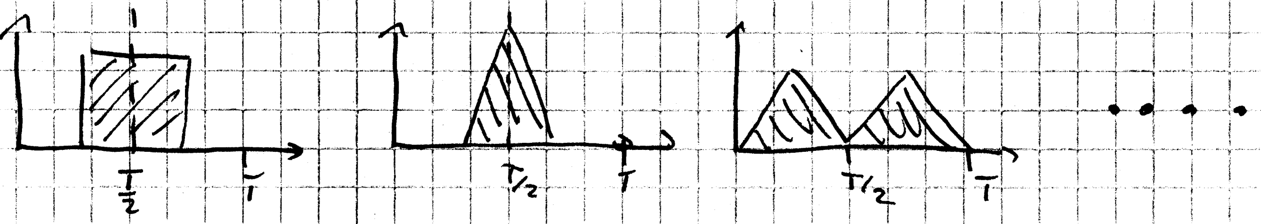


$$t_{L,3} := \frac{\alpha\int_0^T (t-\frac{T}{2})^2 t\,dt}{\alpha\int_0^T (t-\frac{T}{2})^2\,dt} = \frac{}{\frac{1}{3}(t-\frac{T}{2})^3\big|_0^T}$$

$$= \frac{\frac{T^4}{4} - \frac{T^4}{3} + \frac{T^4}{8}}{\frac{1}{3}\left(\frac{T^3}{8}+\frac{T^3}{8}\right)} = \frac{\frac{1}{24}T}{1/12} = \frac{T}{2}$$

→ b.w.

diese Liste läßt sich beliebig fortsetzen, auch sämtliche Stauchungen der angegebenen Funktion führen auf $t_L = \frac{T}{2}$



auch nichtsymmetrische $Q(t)$ könnten gefunden werden, die freien Parameter in den Ansätzen können so gewählt werden, daß jeder Ansatz den gleichen Gesamtabfluß $\int_0^T Q(t)dt \equiv Q_{TOTAL}$ liefert.

FAZIT: Gleiches t_L für zwei Ganglinien heißt noch lange nicht, daß diese Ganglinien irgendwie ähnlich sind, weder quantitativ (siehe I) noch qualitativ (siehe II)

<u>zusatz</u>: ~~qp~~ für die skizzierten Ganglinien erhält man zwar jeweils das gleiche q_{50}-Quantil $(\equiv \frac{T}{2})$, für das q_{80}-Quantil aber gilt:

$$q_{80,2} < q_{80,1} < q_{80,3}$$

der Unterschied in den Ganglinien wird also deutlich!